# Field Goal for the Win

## Objectives

- Students will explore the science and math behind kicking a field goal.
- Students will solve linear equations to determine if the kick is made including interpreting solutions in a context.
- Students will use parametric equations graphically. (Note: no prior exposure to parametric equations is necessary. Students will explore each part separately before putting it all together.)
- Students will use trigonometric ratios to determine horizontal and vertical velocity components. (Note: students do not need any prior exposure to the trigonometric ratios as they are only part of the given equations for horizontal and vertical components of velocity.)

## Vocabulary

- Initial Velocity
- Horizontal
- Vertical

## About the Lesson

- In this activity, students will explore a parametric model of the flight of a ball graphically, numerically and algebraically. As a result, students will:

  Solve the $x(t)$ and $y(t)$ equations separately for values of $t$, then use that value of $t$ to determine a coordinate pair.
- Students should know how to build a table of values given a function and inputs.
- Students should recognize that cosine and sine of an angle yields decimal values.
- Teachers may want to give more guidance regarding parametric equations or trigonometric values using the pre-work handouts provided.

## Activity Materials

- Compatible TI Technologies:

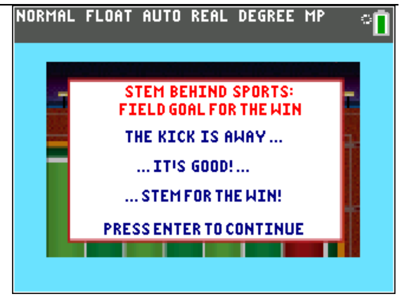
  TI-84 Plus*

  TI-84 Plus Silver Edition*

  TI-84 Plus C Silver Edition

  TI-84 Plus CE

** with the latest operating system (2.55MP) featuring MathPrint™ functionality.*




**Tech Tips:**

- This activity includes screen captures taken from the TI-84 Plus CE. It is also appropriate for use with the rest of the TI-84 Plus family. Slight variations to these directions may be required if using other calculator models.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at [http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials](http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials)
- Any required calculator files can be distributed to students via handheld-to-handheld transfer.

**Lesson Files:**

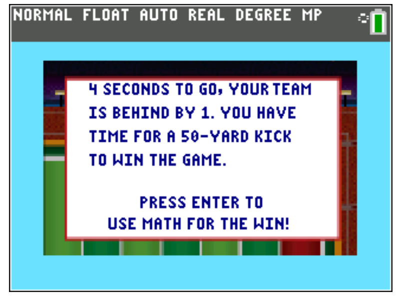
- MGA1_TheKick_Student.pdf
- MGA1_TheKick_Student.doc
- The_Kick.8xp
- Image0.8ca
- Image9.8ca

TI-84 Plus*, and TI-84 Plus Silver Edition* use

- The_Kick.8xp

**Tech tip:** Make sure when sending the program file The_Kick_color.8xp to your TI-84 Plus CE calculators that the program, and image0 and image9 are sent. For TI-84 Plus or TI-84 Plus Silver Edition users, send the program The_Kick.8xp without the image files.

**Introduction**




In this activity, students will use a mathematical model of a field goal kick. Students first explore the model graphically on the calculator to understand how the horizontal and vertical positions are related to time. Students then break the kick into its horizontal and vertical components and use algebra to explore those separately. Finally, students put it all together and determine if their kick is made and wins the game. There are extensions for exploring student created kicks as well as a scenario that involves modeling the defense.

**Teaching Tips and Background:**

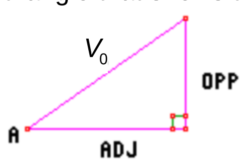
- Use TI-Connect CE to deliver the program and background images to your teacher calculator. Download the latest version at: https://education.ti.com/en/us/software/details/en/CA9C74CAD02440A69FDC7189D7E1B6C2/swticonnectcesoftware
- For extra information about sending programs and images using TI-connect CE download the guidebook at the link below: https://education.ti.com/en/us/guidebook/details/en/9A4FE63E3B054CB49C06B202578AB7FE/ti-connect-ce

- Guide your students on opening and running the program The Kick. While the program is running note the circle in the upper right corner will show whether the program is waiting (white) or computing (yellow). If there is no circle, the program is not running.
- While in the program students will navigate using the arrow keys, number keys, and enter, to control the program. To exit the program and explore choose Option 7 from the submenus. At the end of the activity, make sure students run the program and select Quit, this will set the calculator back to a set of defaults that were changed while in the program.
- Students should have an opportunity (between 2 and 5 minutes) to explore different values for kicks before jumping into question 1. Have them run Option three from the main menu for several different kick lengths, angles, and velocities. Note that kick lengths between 20 and 70 yards, angles should be between 0 and 90 degrees, and kick velocities between 50 and 90 ft/s.
- The mathematical background section is information for you as a teacher. Share with your students as much or as little as you feel comfortable.

**Mathematical background for the model:**

In order to model the motion of the ball during its flight, the model makes use of parametric equations and trigonometric functions. Students do not need prior experience with trigonometry or parametric equations in order to explore this lesson graphically or algebraically. A pre-work exploration is provided in order to familiarize students with the values of the trigonometric functions cosine and sine, for angles between 0 and 90 degrees. This will allow students to be comfortable with the fact that $\cos(43°)$ is just a decimal value between 0 and 1. The pre-work can also be used to show how two values can depend on one independent variable at the same time. This will allow them to see how the horizontal and vertical positions can be shown as functions of time.

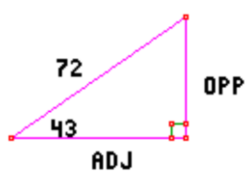
The horizontal and vertical positions of the ball after it is kicked depend on many factors (wind, air resistance, elevation, gravity, etc.). For simplicity and accessibility, in this model we are only including the effect of gravity on the ball. The kick can be broken into its horizontal and vertical components using a triangle that shows the initial velocity as the hypotenuse.




Using the trigonometric ratios:

$\cos(A) = \dfrac{adj}{V_0}$ $\qquad$ $\sin(A) = \dfrac{opp}{V_0}$

$adj = V_0 \cdot \cos(A)$ $\qquad$ $opp = V_0 \cdot \sin(A)$




Horizontal velocity component:

$\cos(43°) = \dfrac{adj}{72}$

$adj = 72 \cdot \cos(43°)$

Vertical velocity component:

$\sin(43°) = \dfrac{opp}{72}$

$opp = 72 \cdot \sin(43°)$

Since $\text{distance} = \text{rate} \cdot \text{time}$, the horizontal distance traveled (position), $x(t)$, is given by : $x(t) = V_0 \cdot \cos(A) \cdot t$ where $V_0$ is the initial velocity, $A$ is the angle of the kick, and $t$, is the time in seconds since the ball was kicked. In this example $x(t) = 72 \cdot \cos(43°) \cdot t$

For the vertical motion, the effect of gravity must be considered. In order to get the vertical position as a function of time, we have to add the effect of gravity to the vertical velocity of the kick. From physics it is known that the effect of gravity on position is $-\dfrac{1}{2} g \cdot t^2$, where $g = 32 \dfrac{\text{ft}}{\text{s}^2}$. The vertical positon of the ball at a time, $t$, can be found using the function $y(t) = V_0 \sin(A) \cdot t - 16 \cdot t^2$. For the above example: $y(t) = 72 \sin(43°) \cdot t - 16 \cdot t^2$

Some information about football that students may need for this activity:

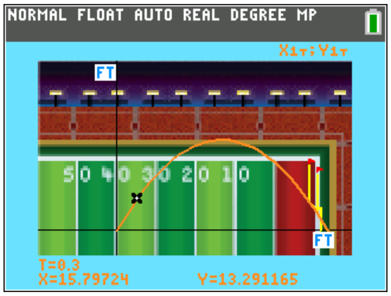
- Remind students there are 3 feet in a yard.
- The crossbar of the goalposts is 10 feet high, and is directly above the back line of the end zone.
- The end zone is 10 yards deep.
- Line of scrimmage is the yard line where the ball is placed before the play begins.
- Professional kickers kick the ball from 7 yards behind the line of scrimmage.
  From this information, when the line of scrimmage is the 33 yard line, a field goal attempt would be a 50 yard kick. (33+10+7=50)

**Get Your Game On**

4 seconds left in the Big Game and the score is 27 to 28, you're behind. A 50-yard field goal wins the game. Your task is to use a mathematical model to demonstrate kicking a field goal to win the game.

## 1. The Kick




Turn on your TI-84, press [prgm], and run the program THEKICK.
Follow the on-screen prompts to the main menu.
Press 1 to see the 50-yard kick.

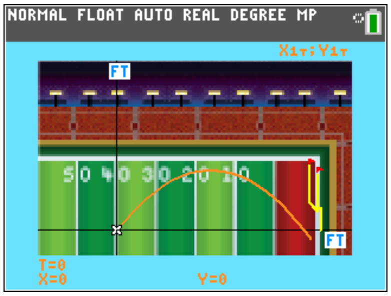
a. Is the field goal good? Write your guess here. (Note: To see the flight of the ball again press [2nd] [mode] (quit) and press [enter] to run the program again.)

**Answer:** Answers will vary

- Students may need to view the kick more than once. Make sure they realize that the orange graph is modeling the flight of the ball fairly accurately.

b. Discuss with your group: When kicking a football, what things affect how far the ball travels downfield and how high the ball reaches?

**Answer:** Factors include: velocity of the kicked ball, angle that the ball is kicked, gravity, wind, air resistance, weather, elevation of the city where game is played, inflation of the ball. In the discussion mention that we are only going to consider velocity, angle, and gravity in this problem.




c. Press [2nd] [mode] (quit). Then run the program again but this time select Option 3 (Custom distance and kick). Use:

Length of kick (Yards): 50 press [enter]
Angle of kick (DEG): 35 press [enter]
Velocity of kick (FT/S) 70 press [enter]
What do you notice about the kick?

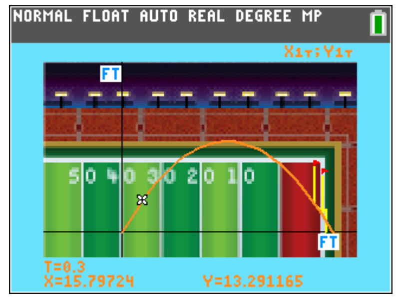
**Answer:** Students will notice that the kick does fall short.

## 1. The Kick (continued)

d. Professional kickers kick the football with a velocity between 70 and 88 feet per second (between 48 and 60 mph). The angle varies between 27 and 43 degrees. Press [enter] to view the Options menu. Using values in these ranges, take two minutes and explore several kicks using Options 2, 3, and 4.

- Students should be encouraged to "play" and experiment with these Options to become familiar with how the angle and velocity affect the flight of the ball – how high and how far the ball travels.

e. Press [enter] to return to the Options menu. Press 6 (Main Menu). Then Option 1. Press [trace]. Look at the information on the screen. Notice the three variables, T, X, and Y. Press the right arrow three times and record the values to the nearest hundredth.




T=____0.3________ X=_____15.78______

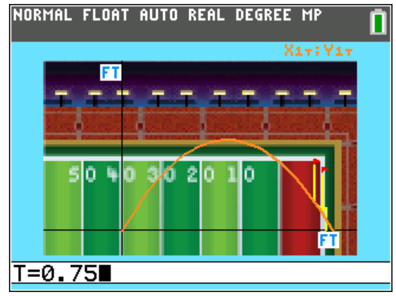
Y=____13.29________

f. Discuss what the values of these variables mean in this problem with your group. Include units in your discussions.

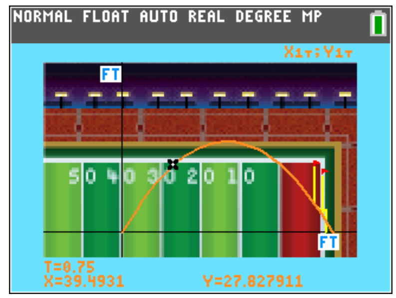
**Answer:** At time $t = 0.3$ seconds, the ball is approximately 15.80 feet downfield in the horizontal direction from where it was kicked, and the ball is approximately 13.29 feet high (in the vertical direction). Some students may not recognize this right away, guide them to a solid understanding of what X, Y, and T are before moving on.

## 1. The Kick (continued)

g. Trace on the graph to where T is three-quarters of a second. (Note: just type 0.75 and press enter) . Discuss what these values mean with your group; include units in your discussion.

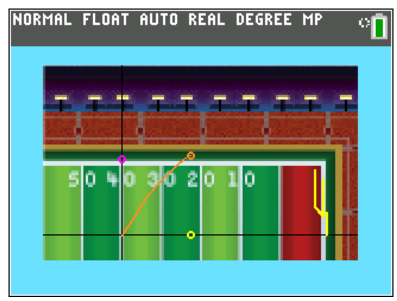



**Answer:** At time = 0.75 seconds, the ball is approximately 39.49 feet downfield in the horizontal direction from where it was kicked, and the ball is approximately 27.83 feet high (in the vertical direction).




- Tech tip: The students need to know that when in trace mode, they can just type the number they want to trace to.

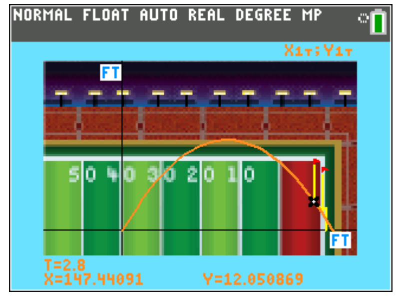
h. Press 2nd mode (quit) and press enter to run the program again but this time select Option 2. (Note: To see this graph again, press enter then choose Option 1, Repeat). Discuss in your groups what the yellow, pink, and orange circles represent.



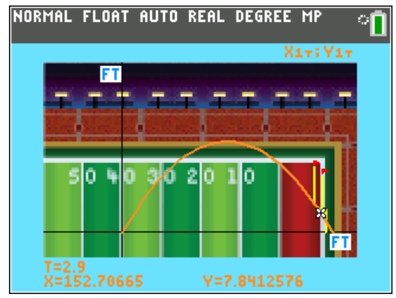

**Answer:** The orange circle is plotting the actual flight of the kicked football. The yellow circle is showing the horizontal distance the ball is traveling downfield. The pink circle is showing the vertical height of the ball.

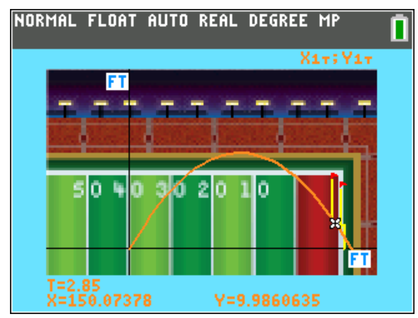
**1. The Kick (continued)**

i. Press $\boxed{\text{enter}}$ to return to the Options menu. Press 7 (Exit and explore). Press $\boxed{\text{trace}}$ and graphically investigate the time when the ball is 150 feet from where it was kicked in the horizontal direction. Between what two values of T is the ball 150 feet downfield? Record the times to the nearest tenth of a second and the corresponding distances to the nearest hundredth of a foot.




**Answer:** The ball is about 150 feet downfield between T=2.8 and 2.9 seconds. At T=2.8 seconds, the ball is 147.44 feet away and at T=2.9 seconds, the ball is 152.71 feet downfield. Students may try values between 2.8 and 2.9 seconds for example, T=2.85 seconds is shown.



**2. Modeling horizontal and vertical motion**

In order to answer question 1i. (When is the ball 150 ft from where it was kicked?) with more precision, we need an expression to model the horizontal distance the ball travels downfield after it is kicked. Call it $x(t)$ (read "x of t"). We also need an expression to model the vertical distance (height) of the ball after it is kicked. Call it $y(t)$ (read "y of t"). Both of these expressions involve ratios that are usually studied in geometry: sine and cosine. You may have been introduced to them in a pre-assignment. For our kick, the football was kicked at an angle of $43°$ with a velocity of 72 ft/s.

The function for horizontal distance traveled downfield is:
$x(t) = 72 \cdot \cos(43°) \cdot t$

The function for vertical distance (height):
$y(t) = 72 \cdot \sin(43°) \cdot t - 16 \cdot t^2$

- We suggest that you explain that the $-16 \cdot t^2$ represents how gravity affects the height of the ball. Without the effect of gravity, the ball would continue rising and never come back to the ground.

**Teaching Suggestion:**

*If your students did the pre-assignment, have them discuss their results in their groups quickly. See the teacher notes for the pre-assignment.*

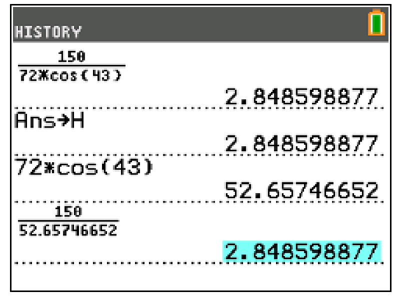
- *If your students did not do the pre-assignment, we suggest that you do the following brief activity with your students so that they get an idea about what sin and cos mean.*

*In class activity for sin and cos.*

- *End the program by pressing* [on] *and selecting quit to return to the home screen on the calculator.*
- *Pair students who are sitting with one another. Give one student the first angle measure and the other student the second angle measure from the list below: (repeat the numbers as needed)*
- *1, 89 10, 80 20, 70 30, 60 40, 50 2, 88*
- *For any student without a partner, assign that student 45.*
- *Ask the students to clear their home screen.*
- *Then ask them to calculate the sine of the angle given to them, then the cosine of the angle given to them. Have each pair of students compare their screens with the sin and cos values. Ask them to look for any pattern they notice.*
- *They should say something like "the sine of one angle is the cosine of the other angle." Some students may notice that the angles measures always add up to 90, that is, the angles are complementary. We also want them to notice and realize that the sine and cosine values are decimals between 0 and 1.*

**2. Modeling horizontal and vertical motion (continued)**

a. Algebraically calculate how long it will take for the ball to travel 150 feet downfield. Store your answer in H.




**Answer:** 2 methods are acceptable. Students may want to compute the decimal approximation for $72 \cdot \cos(43°)$ first and then divide. The preferred method would be not to round until the final calculation.

$$150 = 72 \cdot \cos(43°) \cdot t$$

$$\frac{150}{72 \cdot \cos(43°)} = t$$

$$2.848598877 \approx t$$

OR

$$72 \cdot \cos(43°) \approx 52.65746652$$

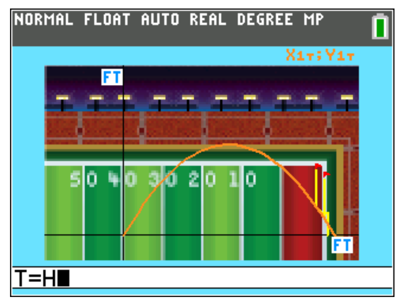
$$52.65746652 \cdot t \approx 150$$

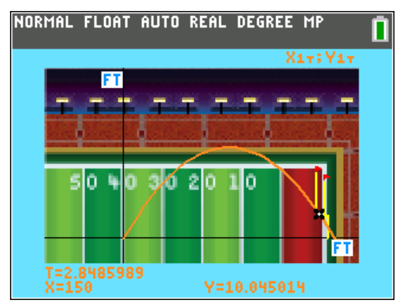
$$t \approx \frac{150}{52.65746652}$$

$$t \approx 2.848598877$$

- Tech tip: Storing a value using the sto→ key will allow students to trace directly to their solution.

b. Press trace and trace to this value for H and explain what the numbers on the screen mean. Is the field goal good? Explain how you know.



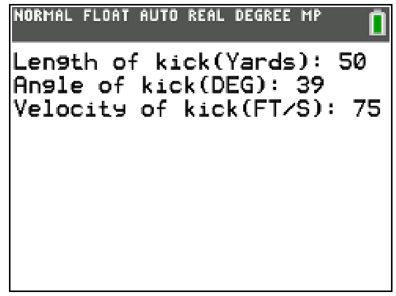




**Answer:** Yes, the field goal is good. At T = H, the ball is exactly 150 feet away (see the x value) in the horizontal direction, which is at the goal post. At that time, the ball is 10.045 feet high (see the y value). Since the crossbar is 10 feet high, the ball does go over the crossbar – but just barely.

- Make sure that students clearly explain why the kick is good, by making reference to the values for x and y and what they mean.

## 3. Application of the model

Run the program THEKICK again and select Option 3. You're going to kick a field goal to win the game. Professional kickers kick the ball with a velocity of about 70 to 88 ft/s (48 to 60 mph) and at an angle that varies between 27 and 43 degrees. Choose your velocity and kick angle and run the program to graph your kick.



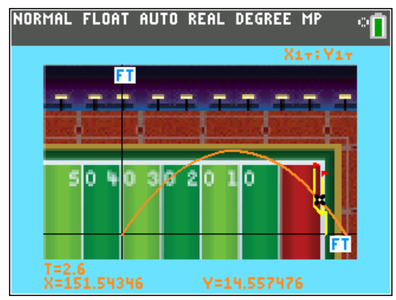

Length of kick (yards): __50__ Angle (degrees): _____39___

Velocity (ft/s): ______75____

a. Use the equations from Question 2 that model the horizontal and vertical positions of the ball to model the actual flight of the ball after it is kicked. Write the equations for your kick below

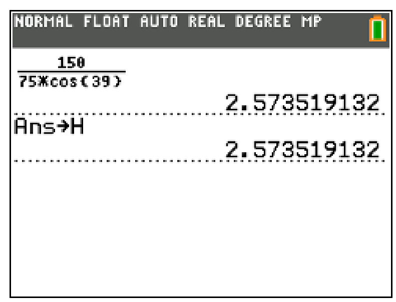
**Answer:** $x(t) = 75 \cdot \cos(39°) \cdot t$ ; $y(t) = 75 \cdot \sin(39°) \cdot t - 16 \cdot t^2$

b. Based on the graph, can you tell if the ball passes above the 10-foot crossbar on the goal posts? How can you tell?




**Answer:** *Yes the ball does pass above the crossbar. At T = 2.6 seconds, the ball is 151 feet downfield at a height of 14.6 feet, well above the crossbar (and past it).*

c. Solve algebraically for when the ball is 50 yards downfield. Use your solution to decide if the field goal is made or not. Explain your response.



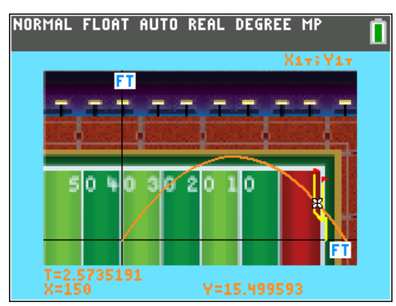

**Answer:**

$$75 \cdot \cos(39°) \cdot t = 150$$

$$t = \frac{150}{75 \cdot \cos(39°)}$$

$$t \approx 2.573619132$$

Yes the field goal is good since the ball is approximately 15.5 feet high which is higher than the crossbar.

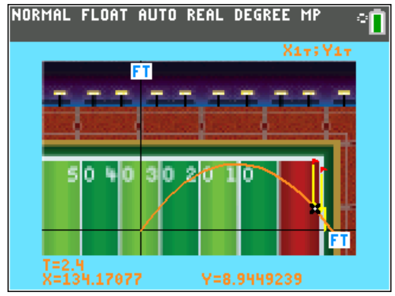


**3. <u>Application of the model (continued)</u>**

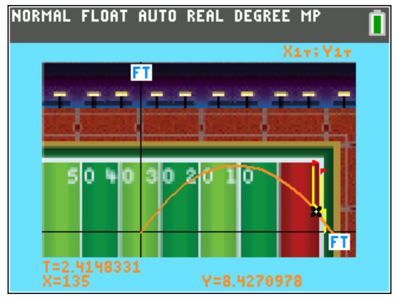
d. Attempt a 45-yard kick with a different angle and velocity and determine graphically if the kick is made using the program. Then use algebra to confirm your answer.

Angle __37____ Velocity ____70______

Made (Y/N) ____N____




**<u>Answer:</u>** The kick is not good because at T=2.4 seconds, the ball is 134 feet downfield (almost 45 yards =135 feet). The height of the ball at that time is less than 9 feet which means it's below the crossbar.




Using algebra:

$$70 \cdot \cos(37°) \cdot t = 150$$

$$t = \frac{150}{70 \cdot \cos(37°)}$$

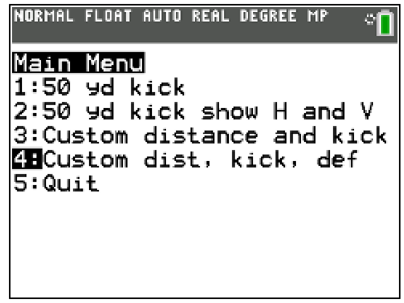
$$t \approx 2.414833055$$

At this value for T, the ball is exactly 45 yards downfield and only 8.4 feet high, -- short of the crossbar. The kick is not good.

## Extensions

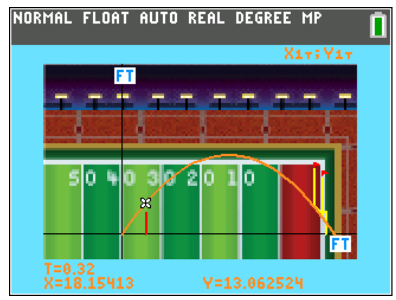
**4. <u>But what about the defense?</u>**

Run the program THEKICK again and select Option 4. You're going to kick a 50-yard field goal to win the game. Professional kickers kick the ball with a velocity of about 70 to 88 ft/s (48 to 60 mph) and at an angle that varies between 27 and 43 degrees. Choose your velocity and kick angle and run the program to graph your kick. The kicker kicks from 7 yards behind the line of scrimmage and the defense typically gets little or no rush (between 0 and 2 yards) and can reach about 8 to 9 feet in the air.




Angle: 39 Velocity: 73 Rush: 1 Reach: 9

Student answers will depend on their scenario. Solutions here are based on a 50-yard field goal, an angle of 39 degrees, a velocity of 73 ft/s, a rush of 1 yard, and reach of 9 feet.




a. Based on your model, will a defender block the kick? Defend your answer graphically and algebraically.

**<u>Answer:</u>**

Using the graph:

Tracing to T=0.32 seconds, the ball is 18 feet downfield ( $7-1=6$ yards ) and at a height of 13.06 feet, which is 4.06 feet above the reach. The defender does not block the kick.

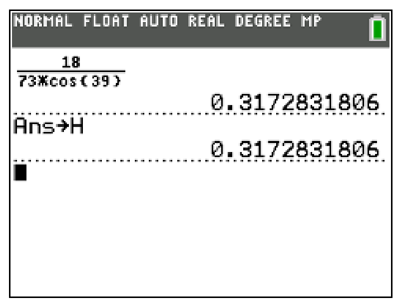
Using algebra:

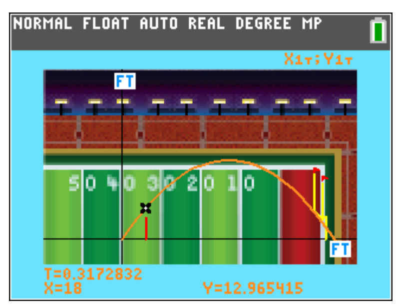
$73\cdot\cos(39°)\cdot t=18$

$t=\dfrac{18}{73\cdot\cos(39°)}$

$t\approx 0.3172832$ seconds

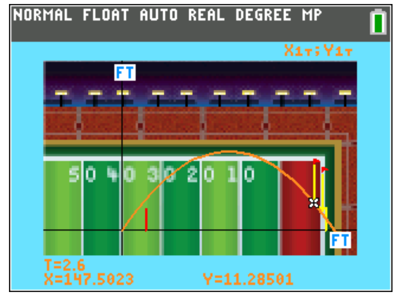
At this time the ball is 18 feet (6 yards) downfield and 12.97 feet in the air, which is above the reach of the defender.

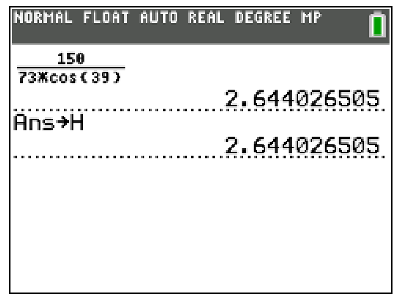


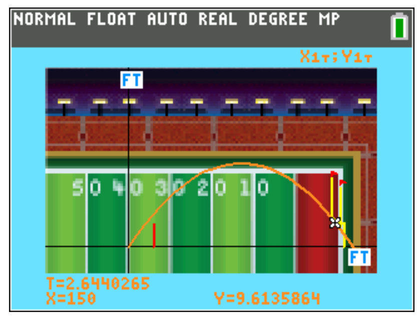



**4. <u>But what about the defense? (continued)</u>**

b. Based on your model, will the ball pass above the 10-foot. crossbar on the goal posts? How can you tell? Defend your answer more than one way.




**<u>Answer:</u>**

Using the graph:

Tracing to T=2.64 seconds, the ball is 149.8 feet downfield (almost 50 yards) and at a height of 9.77 feet, which is below the crossbar. The kick is not good.

Using algebra:

$73 \cdot \cos(39°) \cdot t = 150$

$t = \dfrac{150}{73 \cdot \cos(39°)}$

$t \approx 2.6440265$ seconds

At this time the ball is 150 feet (50 yards) downfield and 9.61 feet in the air, which is below the crossbar. The kick is not good.

c. Attempt the kick with a different angle or velocity and determine algebraically if the kick makes it over the defense and is good. Then use the program to confirm your answer.

Angle ________ Velocity __________ Rush ________ Reach ______

Blocked (Y/N) ________ Made (Y/N) ____________

**<u>Answer:</u>** Answers will vary based upon student input.

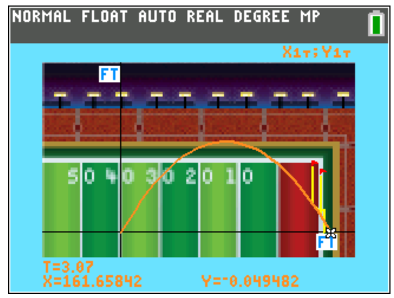
**5. More Extensions**

Graphically and algebraically find:

- When the ball hits the ground
- How far the ball travels horizontally
- The maximum height of the ball

Run the program and select Option 1 (1: 50-yard kick).

a. trace to discover when the ball hits the ground and how far away the ball is from where it was kicked. Write your answers below.



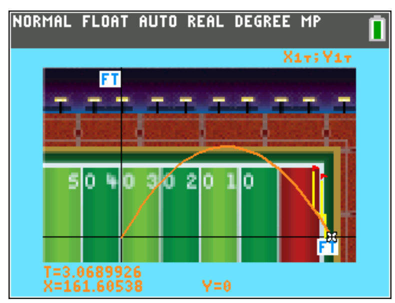

**Answer:** The ball hits the ground between T = 3.0 and 3.1 seconds, to the nearest hundredth of a second, T = 3.07 seconds.

The ball is approximately 161.65842 feet downfield when it hits the ground.

b. Algebraically find when the ball hits the ground and use that value to find how far the ball is downfield when it hits the ground. Include units in your answers and do not round the answers.

**Answer:** To find out when the ball hits the ground, we need to find out when the vertical height of the ball is 0 feet. We need to use the y(t) equation discussed earlier.

Recall that the ball is kicked with an initial velocity of 72 ft/s and at an angle of $43^{\circ}$.




Solve:

$72 \cdot \sin(43°) \cdot t - 16 \cdot t^2 = 0$

Since your students may not have studied factoring yet, it may be useful to show them that the expression can be rewritten using the distributive property. $t(72 \cdot \sin(43°) - 16 \cdot t) = 0$

Since there are two expressions multiplied with an answer of zero, one or the other or both must be zero. $t = 0$ or $72 \cdot \sin(43°) = 0$ Since the first is when the ball is kicked, the second equation must be solved for t when the ball hits the ground.

$72 \cdot \sin(43°) - 16 \cdot t = 0$

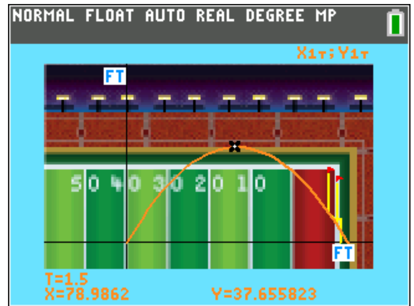
$-16t = -72\sin(43°)$

$t = \dfrac{-72\sin(43°)}{-16}$

$t \approx 3.06899262$ seconds

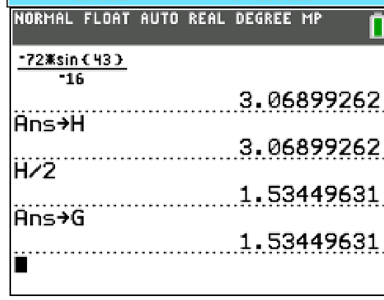
**5. More Extensions (continued)**

c. [trace] to discover when the ball attains its maximum height and what is the maximum height?



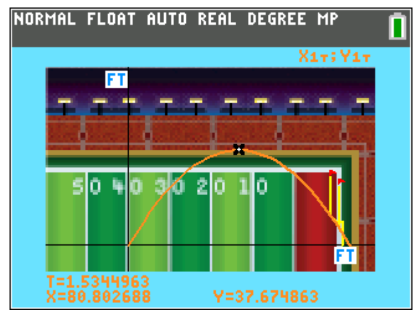

**Answer:** At T = 1.5 seconds, the ball appears to be close to its maximum height, which is 37.655823 feet.

d. Algebraically find when the ball attains its maximum height and use that value to find the maximum height. Include units in your answers and do not round the answers.




**Answer:** To solve this algebraically we need to realize that the graph of the flight of the ball is a parabola. The vertex of the parabola would be the point where the ball attains its maximum height. Because of the symmetry of the shape of the parabola, the vertex should be halfway between where the ball is kicked and where it hits the ground.

If we divide the value we have for when the ball hits the ground by two, we will get the time when the ball is at its maximum height. We stored that into H. We will store half of H into G.




Tracing to T = G we find that the maximum height of the ball is 37.674863 feet.